Math 216 Exam 1 Solutions
Fall 2004

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1. see p. 30 $n \doteq \frac{(z_{\alpha/2})^2 p(1-p)}{D^2} = \frac{1.96^2(.75)(.25)}{(.02)^2}$

$= 1800.75$

$\Rightarrow$ Collect data on 1801 hens.

Conservative approach is ok – $n \doteq \frac{z_{\alpha/2}^2}{4D^2} = \frac{1.96^2}{4(.02)^2} = 2401$

2. Let p = proportion of stations with leaks
We want to test $H_0: p = .40$ vs $H_1: p < .40$

$\hat{p} = \frac{7}{27} = .2593$

(a) With Minitab – Stat > Basic Stat > 1 Prop

• Summarized Data   # of trials = 27
# of events = 7

Click options   Test Proportion = .4
Alternative: Less than

X	N	Sample p	95% UCB	Exact P-value
7	27	.259259	.432302	.095

Since .095 > most reasonable α levels (e.g. .01 or .05), we do NOT have statistically significant evidence to reject H_0, i.e. We do NOT have evidence that the new program results in a significant decrease.

Without MTB – Use LSA. (see p. 22)

$B^* = \frac{B - np_0}{\sqrt{np_0(1-p_0)}} = \frac{7 - 27(.4)}{\sqrt{27(.4)(.6)}} = -1.4928$

Approximate p-value $= P(Z < -1.4928) = .0677448$

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#2 b. YES, Type II error - Fail to reject H_0 when it is false.

#3. Paired Data => Form differences $Z_i = Y_i - X_i$

Y_i = men, X_i = women

(a) Let θ = median of the differences

(b) Nothing said about symmetry (in fact the distribution of the differences is clearly <u>NOT</u> symmetric) so use procedures for the sign statistic - see p. 75 (section 3.6)

Point Estimate: $\hat{\theta} = \text{median}\{Z_i,\ 1 \le i \le n\} = 4.7$

Interval Estimate: Stat > Nonparametrics > 1-Sample Sign

94.26% CI for θ is $(3.1, 10.8)$

$Z^{(4)}, Z^{(11)}$

98.71% C.I. for θ is $(3, 20.4)$

$Z^{(3)}$, $Z^{(12)}$

We are 98.71% confident that the median difference (Men - Women) is in the interval from 3 to 20.4. Since the interval does not include zero, we have significant evidence (at $\alpha = .0129$) that the percentages for men and women differ.

* [Since all differences are greater than zero, any level LCB (e.g. 97.13%, $\alpha = .0287 \Rightarrow (Z^{(4)}, \infty)$) $(3.1, \infty)$ can be used to show then men read significantly more than women in EU countries.

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Alternative solution for #3 (b) : Using C.I based on 1-Sample Wilcoxon (Signed Rank) - Must state that they are assuming symmetry.

Point Estimate: $\hat{\theta} = \text{median}\left\{ \frac{z_i + z_j}{2} \; i \le j \right\} = 6.5$

94.8% C.I. for θ is $(3.6, 14.0)$

#4. (a) Paired Data => Form Differences $z_i = y_i - x_i$

(y_i = Strawberry, x_i = Vanilla)

Let θ = median of the differences

Nothing said about symmetry (In fact the distribution of the differences is clearly <u>NOT</u> symmetric) so use sign test.

$H_0: \theta = 0$ vs $H_1: \theta \neq 0$

STAT > Nonparametrics > 1-Sample Sign

N	Below	Equal	Above	P	Median
12	2	5	5	.4531	0

STAT > Nonparametrics > 1-Sample Wilcoxon (Must assume symmetry)

N	N for test	Wilcoxon	P	Estimated Median
12	7	21.0	.272	5

Since .4531 (or .272) > .05, we cannot reject H_0. i.e., these data do <u>NOT</u> suggest that there is a significant difference in calories per serving of strawberry and vanilla yogurt.

Ideally students will comment on the large number of ties for this small data set.

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#4 (b) Assume symmetry => Use 1 Sample Signed Rank Procedures

We want to test $H_0: \theta = 120$ vs $H_1: \theta \neq 120$, where θ is the median number of calories from a serving of vanilla yogurt.

C.I. for θ is based on Walsh averages (see p. 56)

$$C_\alpha = \frac{n(n+1)}{2} + 1 - t_{\alpha/2}$$

From Table A.4. Use $\alpha/2 = .026 \Rightarrow \alpha = .052$

$$C_{.052} = \frac{12(13)}{2} + 1 - 64 = 15$$

$\Rightarrow$ confidence level = 94.8%

94.8% CI for θ is $\left(W^{(15)}, W^{(64)}\right)$

$$(130, 190)$$

We are 94.8% confident that the median number of calories for a serving of vanilla yogurt is between 130 and 190. Since 120 is NOT in this interval we can reject H_0 and conclude that θ is significantly different from 120 or the claim is incorrect.

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#5. (a) Any effect => Use Kolmogorov-Smirnov

H_0: $F(t) = G(t)$, for every t

H_1: $F(t) \neq G(t)$, for at least one t.

F nonconfined, G confined.

Ordered Values

Nonconfined (F_m)	Confined (G_n)	$\lvert F_m(t) - G_n(t) \rvert$
	9.0	$\lvert 0 - 1/10 \rvert = 1/10$
	9.2	$\lvert 0 - 2/10 \rvert = 2/10$
	9.3	$\lvert 0 - 3/10 \rvert = 3/10$
	9.5	$\lvert 0 - 4/10 \rvert = 4/10$
9.6	9.6	$\lvert 1/10 - 5/10 \rvert = 4/10$
	9.7	$\lvert 1/10 - 6/10 \rvert = 5/10$
	9.9	$\lvert 1/10 - 7/10 \rvert = 6/10$
10.3	10.3	$\lvert 2/10 - 8/10 \rvert = 6/10$
10.4, 10.4	10.4	$\lvert 4/10 - 9/10 \rvert = 5/10$
10.5		$\lvert 5/10 - 9/10 \rvert = 4/10$
10.7, 10.7		$\lvert 7/10 - 9/10 \rvert = 2/10$
10.9	10.9	$\lvert 8/10 - 1 \rvert = 2/10$
11.1		$\lvert 9/10 - 1 \rvert = 1/10$
11.2		$\lvert 1 - 1 \rvert = 0$

Test Stat: $J = \frac{mn}{d} \max \lvert F_m(t) - G_n(t) \rvert$

$$= \frac{10(10)}{10} \cdot \frac{6}{10} = 6$$

p-value $= P(J \geq 6) = .0524$ (Table A.10 p. 609)

Since $.0524 > .05$, we cannot reject H_0. According to the Kolmogorov-Smirnov test, sensory deprivation does not have a significant effect. However, the p-value is very close to .05.

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#5 (b) I would recommend the Jackknifing procedure to test $H_0: \gamma^2 = 1$ vs $H_1: \gamma^2 \neq 1$ because there is no information to suggest that $\theta_1 = \theta_2$ (the two medians, confined and nonconfined) are equal).

(c) Look for shift difference to the left $\Rightarrow$ Use one sided Wilcoxon Rank Sum (2 sample procedure)

$H_0: \theta_1 = \theta_2$

vs $H_1 \quad \underset{\text{nonconfined}}{\theta_1} > \underset{\text{confined}}{\theta_2}$

Test Stat: $W = 140.5$

p-value (adjusted for ties) $= .004$

Since $.004 < .05$, we reject H_0 and conclude that frequencies are lower for the confined group. i.e. the sensory deprivation does have a significant location effect.

X F Non

Y G Con